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Packet Transport on Complex Networks With and Without Priority-Based Protocol

K.-H. Kim, B. Kahng* and D. Kim
Department of Physics and Astronomy, Seoul National University, Seoul 151-747

We study packet transport on scale-free networks with N nodes. At each time step, pN packets are created in the system and sent to randomly selected targets. We first review previous results on the jamming transition occurring at the critical point p_c and models to enhance the transport efficiency. Next, we introduce a priority-based protocol and study the packet transport based on it. We consider two cases: (i) A fraction f of generated packets are tagged as priority-assigned; (ii) the packets are all of the same type. For case (i), packets with priority are sent first, compared with those without priority. We find that the jamming transition point is extended for the packets with priority; however, it is reduced for those without priority.

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I. INTRODUCTION

Recently, extensive attention has been focused on complex networks in diverse disciplines such as physics, biology, sociology, economics, and so on. A complex network consists of vertices and edges, which represent elements and interactions between them in complex systems, respectively. For some systems, edges can be physical wires connecting two elements. For example, the Internet is a complex network in which vertices are autonomous systems (AS) or routers and edges are physical wires between AS's. Transport problems on complex networks, such as signal transport in neural networks [1], drivers on road networks [2], data packet transport in the Internet [3], and so on have drawn much attention. This is so because emergent phenomena arise in such systems, created from cooperation between elements.

One of the interesting issues in transport on complex networks is efficiency in transport. This can be the problem of how to improve the amount of transport with the least cost on a given network. To achieve this goal, much research has focused on how to find efficient pathways between sources and targets, which is equivalent to the problem of how to design an efficient routing strategy. Besides, researchers have been interested in designing a network structure towards maximizing the amount of transportation. From the theoretical perspective, the transport problem is interesting, since it exhibits a nontrivial phase transition from free-flow state to congested state as the number of packets generated in the system increases. Moreover, it shows a self-organized pattern near the critical point, where the traffic flow follows a $1/f$-type temporal pattern because of nontrivial traffic congestion and release [4,5].

*E-mail: kahng@phya.snu.ac.kr

II. TRAFFIC MODELS

Ohira and Sawatari [6] built a frame for modeling such a phenomenon. The model introduced hosts and routers which are located at the boundary and bulk, respectively, of a two-dimensional regular lattice. They play roles of generating and receiving packets and delivering, respectively. The routers are capable of queueing an unlimited number of packets. At each time step, packets are generated at randomly selected hosts and are delivered towards randomly assigned targets under the routing strategy, which is based on the shortest path between the source and target. When the number of shortest paths is more than one, the routing strategy chooses one of them deterministically or probabilistically, depending on the traffic on the nearest neighbors of the router that is located on the shortest pathways. As the generating rate p increases, the traffic exhibits a phase transition from a free-flow to a congested phase at a critical point p_c, where the congestion is measured in terms of the average travel time of packets. It was noted that the critical point p_c was sensitive to the router's strategy. From the perspective of transport efficiency, it is interesting to design the routing strategy to enhance the critical point p_c.

Study of the packet transport problem on complex networks was initiated by Goh *et al.* [7]. Their study focused on the heterogeneity of traffic load of each node in a complex network when every pair of nodes sends and receives a unit packet and the packet travels along the shortest path between them. Such a load is closely related to the betweenness centrality.

The extension of the Ohira and Sawatari model to a complex network such as a scale-free tree, randomly grown tree, and directed tree was carried out by Tadić *et al.* [5,8–10]. They modified the rule of packet transport introduced above in several ways: limited queue size, last-in-first-out queues, alternative routing strategy based on local search, *etc.* Such models are able to reproduce the behaviors observed in the Internet traffic, such as the broad distribution of travel times of packets, the transit times of packets following a power-law distribution, *etc.*

Much research has followed the direction of how to improve the traffic efficiency under a given network structure. When a packet is sent from one node to another in the network, it is usual to route packets along the shortest path. Undoubtedly, that is the best way when the number of packets is relatively small. However, when the number increases, a traffic jam can occur, since certain nodes become overloaded. In this case, the routing protocol based on the shortest path is not the best way any more. In particular, this behavior can be worse in scale-free networks at the hub. To resolve this difficulty, Sreenivasan *et al.* [11] suggested a hub avoidance protocol (HA) where traffic is centralized. Their simulation result shows that such a protocol is effective. Similarly, Yan *et al.* [12] proposed another routing protocol, which finds the path that minimizes the so-called 'efficient path' defined below:

$$L(\beta) = \sum_{i=0}^{n-1} k(x_i)^{\beta}, \qquad (1)$$

where n is the path length between given source and target nodes, $k(x_i)$ is the degree of node i on a certain path, and β is a tuning parameter. The intention of this protocol is similar to that of the HA protocol, avoiding overloaded nodes which have large degree. Danila *et al.* [13] also proposed a new heuristic algorithm called the optimal routing protocol, which is also similar to those mentioned above but is more efficient. The algorithm intends to make the load distribution flat, hence decentralizing the concentration of packets at few nodes to other nodes.

Echenique *et al.* [14] studied the congestion transition depending on routing protocol. They introduced a new routing protocol called the 'traffic-aware' protocol, which contains a tuning parameter. If a node i creates a packet whose destination is node t, then the packet is sent to a neighbor node j of the node i that is selected by minimizing the effective distance δ_j between j and t, defined as:

$$\delta_j = hd_j + (1-h)c_j, \quad j = 1, \ldots, k_i, \qquad (2)$$

where d_j is the shortest path length between nodes j and t, c_j is the number of packets in the queue of node j, and h is a tunable parameter in the range $0 \leq h \leq 1$. Notice that we recover the shortest path routing protocol when $h = 1$, but if $h \neq 1$, packets travel longer than the shortest path, but have less jammed paths. Hence, the total travel time of the packet can be reduced. Let $A(t)$ be the number of packets in all queues at time t. They define the order parameter of the jamming transition as $\rho = \lim_{t\to\infty}(A(t+\tau) - A(t))/(\tau p)$, where τ is the observation time. It is observed that the order parameter exhibits a change from a continuous to a discontinuous phase transition as h changes from $h = 1$ to $h \neq 1$. Moreover, the critical point for $h \neq 1$ is higher than that for $h = 1$, implying that the network capacity is larger than that with shortest-path routing.

A routing protocol based on the shortest paths or on minimizing the efficient path needs information on global network topology, which requires huge computing time. Thus, several effective local search algorithms have been studied to reduce the computing time. The traffic-aware algorithm proposed by Echenique *et al.* [14] is one of these. Tadić *et al.* [5,8–10] and Clauset and Moore [15] studied a local search routing protocol. This algorithm is basically a random-walk algorithm in which a packet moves to one of its neighboring nodes randomly; however, it searches up to the next nearest neighbors (NNN). If the target is located in NNN, then the packet moves towards the target; otherwise, it moves to a randomly selected nearest neighbor. They observed that the local NNN search improves the traffic efficiency compared with the shortest-path routing protocol, and the first-order phase transition occurs with this protocol. However, it is noteworthy that the NNN search routing protocol is a static routing protocol, since it does not consider the information on node congestion.

The design of a network structure to improve the efficiency of packet transport is also an important issue. Let us mention the first work introduced by Guimerà *et al.* [16]. They found that a homogeneous-isotropic structure is optimal when the number of packets is large; however, a star-like structure, an extreme of heterogeneous networks, is optimal when the number of packets is small. A similar result was obtained by Tadić *et al.* [5,8–10]. Toroczkai and Bassler [17] suggested that the emergence of a scale-free network in the real world was rooted from the efficiency of packet transport. They performed the simulation of packet transport on two different types of network structure, one random and the other scale-free, finding that the scale-free networks are less prone to jamming than random networks.

III. PRIORITY-BASED ROUTING PROTOCOL

Here, we introduce a new type of routing protocol based on priority. For simplicity, we consider the case where there are two types of packets, with and without priority. Each packet is tagged as either priority-assigned or not when it is created. Packets with priority may be

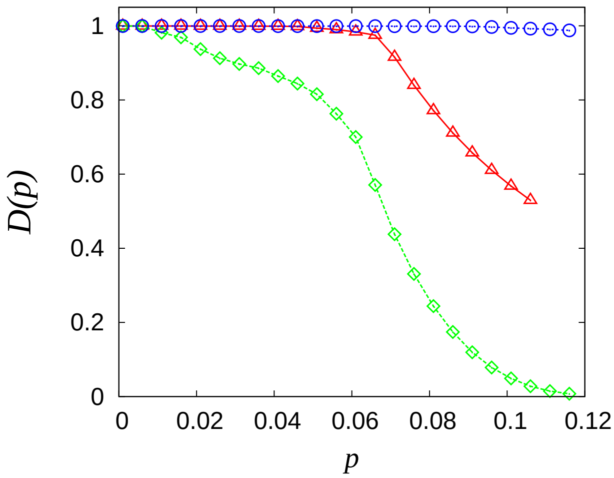

Fig. 1. Delivery fraction $D(p)$ for packets with (∘) and without ( ) priority when the system adapts to the priority-based protocol, and those (△) when the system does not adapt. The priority assignment fraction f is given as 0.5. All data points are averaged over 10 configurations.

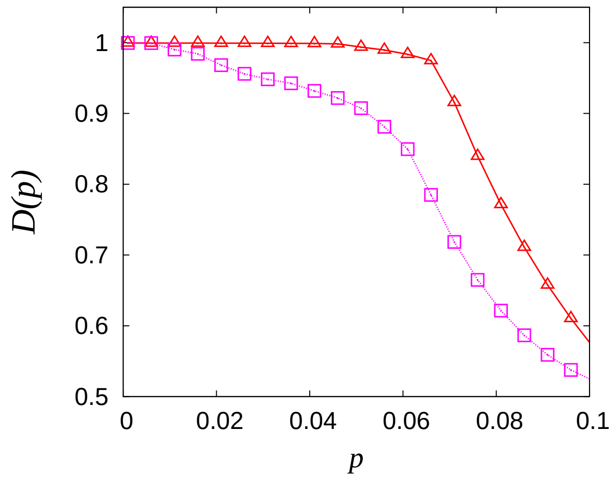

Fig. 2. Delivery fraction $D(p)$ for all packets when the system does ( ) and does not (△) adapt to the priority-based protocol. The priority assignment fraction f is given as 0.5. All data points are averaged over 10 configurations.

regarded as paid packets when we download them from a website. Packets with priority are treated first in a queue, before those without priority. We compare the transport efficiency in this case with the case without priority.

To be specific, we implement the routing protocol by combining the priority-based protocol for determining which packet moves first and the Dijkstra algorithm [18] for determining the node to move to in a further step. The Dijkstra algorithm is a method to find paths with minimum cost between two different nodes in weighted networks. We regard the number of packets accumulated in each queue as the cost. Thus, using the Dijkstra algorithm, we can find the path with least accumulated load and send a packet along that path. We define a new effective distance δ_{it} between a node i and target t as

$$\delta_{it} = d_{it} + \sum_{k \in \mathrm{path}} c_k, \qquad (3)$$

where d_{it} is the distance between node i and target t, and c_k is the number of packets accumulated in each node k on the path from node i to target t, excluding the two end nodes. We route packets along the path that minimizes the effective distance at each time. Note that c_k can change with time. Thus, the path can change from time to time. Another point that we notice is that in the Echenique *et al.* [14] case, they check the queue of the nearest neighbor nodes; however, we here consider the congestion of a whole network. Thus, our case is a generalization of the Echenique case. As for the priority-based protocol, we assign each packet one of two types of protocol, namely with and without priority. Packets with priority are treated first in the routing protocol, before those without priority.

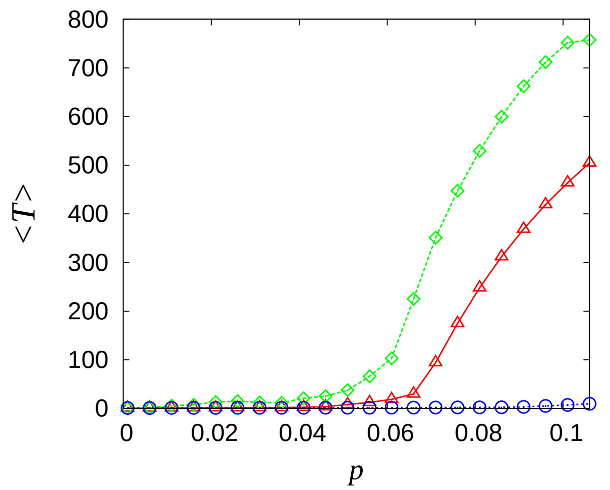

Fig. 3. Mean travel time $\langle T \rangle$ for packets with (∘) and without ( ) priority when the system adapts to the priority-based protocol, and those (△) when the system does not. The priority assignment fraction f is given as 0.5. All data points are averaged over 10 configurations.

IV. NUMERICAL RESULTS

We simulate the packet transport under the dynamic rule below on undirected scale-free networks generated by the static model [7]. The number of nodes is $N = 1000$ and the degree exponent $\gamma = 2.4$. At each time step, every node creates a packet with probability p, whose target is chosen randomly. A fraction f of the packets are tagged as priority-assigned, and the others not. A packet is accumulated in a queue before moving to a next node, and the queue size is unlimited. If there is more than one packet with priority in the queue, then the FIFO rule is applied among them. The Dijkstra algorithm is applied to choose the node to which to advance. This procedure is repeated for as many as $10,000$ steps.

To characterize the jamming transition, the delivery fraction D is defined below:

$$D = \lim_{t\to\infty} \frac{1}{Np(t-t_0)} \int_{t_0}^{t} \lambda(t')dt', \quad (4)$$

where $\lambda(t)$ is the number of delivered packets at time t. If the system is in the free-flow state, $D = 1$, since packets are sent to their destination without being congested. However, if the system is in a congested state, $0 < D < 1$, corresponding to the state of being partially congested, or $D = 0$, corresponding to the perfectly congested state. We can define another quantity, which plays a role of order parameter, called the average accumulation rate in Ref. [10] or the jamming coefficient in Ref. [16]:

$$J = 1 - \left\langle \frac{N_{\rm delivered}}{N_{\rm created}} \right\rangle, \quad (5)$$

where $N_{\rm delivered}$ and $N_{\rm created}$ are the numbers of packets successfully delivered to the target and created, respectively and the bracket denotes the time average. The delivery fraction D is related to the average accumulation rate or jamming coefficient J as follows:

$$D = 1 - J. \quad (6)$$

We also measure the average travel time $\langle T\rangle$, defined as

$$\langle T\rangle = \left\langle \frac{\sum_{i\in \rm path} t_q(i)}{{\rm distance}(s,t)} + 1 \right\rangle, \quad (7)$$

where $t_q(i)$ is the time duration which the packet spent in queue i, and distance(s,t) is the chemical distance between source and target. The average is taken only for delivered packets. Actually, it is another order parameter, since $\langle T\rangle = 1$ if the system is in the free-flow state because of $t_q(i) = 0$; and $\langle T\rangle > 1$ if the system is in the congested state, since $t_q(i) > 0$ for certain nodes on the path.

Figure 1 shows that the system undergoes the jamming transition around the critical point $p_c^0 \simeq 0.06$ when the priority-based protocol is not used. However, when the priority-based protocol is used, the transition point occurs at larger p for the packets with priority, but it occurs at smaller p for those without priority. Figure 3 shows similar results. In our simulations, the fraction f of packets with priority was given as 0.5. However, the whole network performance with the priority-based protocol becomes worse in comparison with that without it, as shown in Figure 2.

V. CONCLUSION

In summary, we have compared packet transport on scale-free networks with and without a priority-based protocol. We have also considered a new dynamic routing strategy, in contrast to other static routing strategies. We have shown that there is an improvement of transport capacity for priority-assigned packets with the priority-based protocol, compared with that without the priority-based protocol, by using various order parameters. More investigations including transport on various network structures other than scale-free networks and measurements of other quantities remain for future work.

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REFERENCES

[1] L. C. Jia, C. R. Han, C. H. Chang, M. T. Tsai, Y. S. Chou, P.-Y. Lai and C. K. Chan, J. Korean Phys. Soc. **50**, 207 (2007).
[2] H.-J. Youn, H. Jeong, F. Roth, M. Silver, M.-H. Clutier and P. Ittzes, J. Korean Phys. Soc. **48**, S217 (2006).
[3] B.-H. Wang and T. Zhou, J. Korean Phys. Soc. **50**, 134 (2007).
[4] R. V. Sole and S. Valverde, Physica A **289**, 595 (2001).
[5] B. Tadić, S. Thurner and G. J. Rodgers, Phys. Rev. E **69**, 036102 (2004).
[6] T. Ohira and R. Sawatari, Phys. Rev. E **58**, 193 (1998).
[7] K.-I. Goh, B. Kahng and D. Kim, Phys. Rev. Lett. **87**, 278701 (2001).
[8] B. Tadić and S. Thurner, Physica A **332**, 566 (2004).
[9] B. Tadić and G. J. Rodgers, Adv. Complex Systems **5**, 445 (2002).
[10] B. Tadic, G. J. Rodgers and S. Thurner, Int. J. Bifur. Chaos 17, 2363 (2007).
[11] S. Sreenivasan, R. Cohen, E. Lopez, Z. Toroczkai and H. E. Stanley, Phys. Rev. E **75**, 036105 (2007).
[12] G. Yan, T. Zhou, B. Hu, Z.-Q. Fu and B.-H. Wang, Phys. Rev. E **73**, 046108 (2006).
[13] B. Danila, Y. Yu, J. A. Marsh and K. E. Bassler, Phys. Rev. E **74**, 046106 (2006).
[14] P. Echenique, J. Gomez-Gardenes and Y. Moreno, Europhys. Lett. **71**, 325 (2005).
[15] A. Clauset and C. Moore, cond-mat/0309415 (2003).
[16] R. Guimerà, A. Diaz-Guilera, F. Vega-Redondo, A. Cabrales and A. Arenas, Phys. Rev. Lett. **89**, 248701 (2002).
[17] Z. Toroczkai and K. E. Bassler, Nature (London) **428**, 716 (2004).
[18] T. H. Cormen, C. E. Leiserson, R. L. Rivest and C. Stein, *Introduction to Algorithms*, 2nd ed. (MIT Press and McGraw-Hill, 2001).
[19] D. Gross and C. M. Harris, *Fundamentals of Queuing Theory*, 3rd ed. (Wiley, 1998).